Benha University Faculty of Engineering – Shoubra Industrial Engineering Department Duration: 2 hours		Final Exam Course: Mathematics 4 Code: EMP 202 Date : December 23, 2017

The exam consists of one page   No. of questions: 4   Answer **All** questions   Total Mark: 40

Question 1

Solve the following equations : **12**

(a) $y` - y = x\, e^{x}$ (b) $(y - 2\sin x)\mathrm{dx} + (\mathrm{x} + \cos \mathrm{y})\mathrm{dy} = 0$

(c) $y`` - 4y` + 4y = e^{3x}$    (d) $y`` + 4y = 4 + 5\cos 3x$

Question 2

(a)Find the L.T of : (i) $\mathrm{f(t)} = 2 + \mathrm{e}^{3\mathrm{t}} + \cosh 2\mathrm{t}$ (ii) $\mathrm{f(t)} = \mathrm{t} + \sin \mathrm{t} + \mathrm{e}^{2\mathrm{t}}.\cos \mathrm{t}$ **2**

(b)Find the inverse L.T of : (i) $\mathrm{F(s)} = \frac{1}{\mathrm{S}^2} + \frac{\mathrm{s}}{\mathrm{s}^2+4}$ (ii) $\mathrm{F(s)} = \frac{\mathrm{s}}{\mathrm{S}^2-3\mathrm{s}+2}$ **2**

(c)By L.T, solve the equation : $\mathrm{y}`` - 2\mathrm{y}` + \mathrm{y} = \mathrm{e}^{\mathrm{t}}$, y(0) = 0 , y`(0) = 1. **4**

Question 3

(a) Using the bisection method, find a root to the equation : $2^{\mathrm{x}} - 2\mathrm{x} - 0.5 = 0$ **4**

in the interval [0, 1], number of iterations is 3.

(b)Find the curve $y = a\, x^{b}$ and the curve $y = a + bx + cx^2$ that fit the data : **4**

(1, 3), (2, 4), (4, 7), (5, 12), (6, 16)

(c)Write the table of differences of the data: (2, 3), (3, 5), (4, 8), (5, 15). **6**

Also, find the value of y at x = 2.5

Question 4

(a)Find the integrals : (i) $\int_0^1 \frac{1}{\sqrt{x^3+x}}\,dx$ (ii) $\int_1^{\infty} \frac{x^2}{1+x^4}\,dx$ **4**

(b)Find $f`(2)$ where $f(x) = \begin{cases} x^3 - 1, & x > 2 \\ 3^x - 1, & x \le 2 \end{cases}$ and h = 0.1 **2**

Good Luck   *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a) It is linear. Then $\rho = e^{\int -dx} = e^{-x}$

Then the solution is : $y.e^{-x} = \int e^{-x}.x\,e^{x}dx = \int x\,dx = \frac{1}{2}x^2 + c$

(b)It is exact because $p_y = 1 = q_x$.

Then $\int (y - 2\sin x)dx = xy + +2\cos x$ and $\int (x + \cos y)dy = xy + \sin y$

Then the solution is : $xy + 2\cos x + \sin y = c$

(c)The A.E is $m^2 - 4m + 4 = 0$. Then $m = 2, m = 2$

Then $y_{c.f} = A\,e^{2x} + B\,x\,e^{2x}$

Also, $y_{P.I} = \frac{1}{D^2 - 4D + 4}e^{3x} = \frac{1}{9 - 12 + 4}e^{3x} = e^{3x}$

The solution is : $y = y_{c.f} + y_{P.I}$

(d) The A.E is $m^2 + 4 = 0$. Then $m = 2i, m = -2i$

Then $y_{c.f} = A\cos 2x + B\sin 2x$

Also, $y_{P.I} = \frac{1}{D^2+4}(4 + 5\cos 3x) = \frac{1}{D^2+4}4 + \frac{1}{D^2+4}5\cos 3x$

$$= \frac{4}{0+4} + \frac{5}{-9+4}\cos 3x = 1 - \cos 5x$$

The solution is : $y = y_{c.f} + y_{P.I}$

--*12-Marks*

Answer of Question 2

(a)(i) $F(s) = \frac{2}{S} + \frac{1}{S-3} + \frac{s}{s^2-4}$ (ii) $F(s) = \frac{1}{S^2} + \frac{1}{S^2+1} + \frac{s-2}{(s-2)^2+1}$

--*2-Marks*

(b) (i) $f(t) = t + \cos 2t$

(ii) $F(s) = \frac{s}{S^2 - 3s + 2} = \frac{A}{s-2} + \frac{B}{s-1} = \frac{2}{s-2} - \frac{1}{s-1}$. Then $f(t) = 2e^{2t} - e^{t}$

--*2-Marks* (c)Since $L\{y'' - 2y' + y\} = L\{e^t\}$, $y(0) = 0$, $y'(0) = 1$.

Then $(s^2 Y - sy(0) - y'(0)) - 2(sY - y(0)) + Y = \frac{1}{s-1}$

From the given condition : $s^2 Y - 0 - 1 - 2Y - 0 + Y = \frac{1}{s-1}$

Then $(s^2 - 2s + 1)Y = \frac{1}{s-1} + 1 = \frac{s}{s-1}$

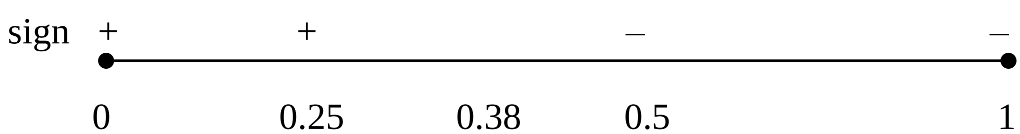
Then $Y = \frac{s}{(s-1)(s^2-2s+1)} = \frac{s}{(s-1)^3} = \frac{s-1+1}{(s-1)^3} = \frac{1}{(s-1)^2} + \frac{1}{(s-1)^3}$

Then $y(t) = t.e^t + \frac{1}{2} t^2 . e^t$

---*4-Marks*

<u>Answer of Question 3</u>

(a) $f(x) = 2^x - 2x - 0.5 = 0$



Iteration	[a, b]	x_n	$f(x_n)$	sign
1	[0, 1]	0.5	– 0.09	–
2	[0, 0.5]	0.25	0.19	+
3	[0.25, 0.5]	0.38		

Then $x^* = 0.38$

---*4-Marks*

(b) The curve is : $y = a\, x^b = 2.55\, x^{0.91}$

The parabolic curve is : $y = a + bx + cx^2 = 3.86 - 1.23x + 0.55x^2$

---*4-Marks*

(c)Since h = 1, then the table of finite differences is:

x	y	Δ	Δ^2	Δ^3
2	3			
		2		
3	5		1	
		3		3
4	8		4	
		7		
5	15			

The forward formula is:

$$y = f(x) = 3 + \frac{2}{1!}(x-2) + \frac{1}{2!}(x-2)(x-3) + \frac{3}{3!}(x-2)(x-3)(x-4)$$

When x = 2.5, y = f(2.5) = 4.06

---*6-Marks*

<u>Answer of Question 4</u>

(i) $\int_0^1 \frac{1}{\sqrt{x^3+x}}\,dx = \int_{0.1}^1 \frac{1}{\sqrt{x^3+x}}\,dx = 1.22$

(ii) $\int_1^\infty \frac{x^2}{1+x^4}\,dx = \int_0^1 \frac{1}{1+y^4}\,dy = 0.87$

---*4-Marks*

(b)From $f(x) = \begin{cases} x^3 - 1, & x > 2 \\ 3^x - 1, & x \le 2 \end{cases}$ and h = 0.1

Then $f`(2) = \frac{f(x+0.1)-f(x-0.1)}{2(0.1)} = \frac{[(2.1)^3-1]-[3^{1.9}-1]}{2(0.1)} = \frac{8.26-7.06}{0.2} = 6$

---*2-Marks*

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